

**Alcuni sviluppi di McLaurin notevoli**

(si sottintende ovunque che i resti sono trascurabili per  $x \rightarrow 0$ )

$e^x$	$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + o(x^n)$	$= \sum_{k=0}^n \frac{x^k}{k!} + o(x^n)$
$\sinh x$	$= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$	$= \sum_{k=0}^n \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2})$
$\cosh x$	$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$	$= \sum_{k=0}^n \frac{x^{2k}}{(2k)!} + o(x^{2n+1})$
$\tanh x$	$= x - \frac{1}{3}x^3 + \frac{2}{15}x^5 + o(x^6)$	
$\ln(1+x)$	$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1} \frac{x^n}{n} + o(x^n)$	$= \sum_{k=1}^n (-1)^{k-1} \frac{x^k}{k} + o(x^n)$
$\sin x$	$= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$	$= \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2})$
$\cos x$	$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$	$= \sum_{k=0}^n (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n+1})$
$\tan x$	$= x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + o(x^6)$	
$\arcsin x$	$= x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \dots + \left  \binom{-1/2}{n} \right  \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$	$= \sum_{k=0}^n \left  \binom{-1/2}{k} \right  \frac{x^{2k+1}}{2k+1} + o(x^{2n+2})$
$\arccos x$	$= \frac{\pi}{2} - \arcsin x$	
$\arctan x$	$= x - \frac{x^3}{3} + \frac{x^5}{5} + \dots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$	$= \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{2k+1} + o(x^{2n+2})$
$(1+x)^\alpha$	$= 1 + \alpha x + \binom{\alpha}{2} x^2 + \binom{\alpha}{3} x^3 + \dots + \binom{\alpha}{n} x^n + o(x^n)$	$= \sum_{k=0}^n \binom{\alpha}{k} x^k + o(x^n)$
$\frac{1}{1+x}$	$= 1 - x + x^2 - x^3 + x^4 + \dots + (-1)^n x^n + o(x^n)$	$= \sum_{k=0}^n (-1)^k x^k + o(x^n)$
$\frac{1}{1-x}$	$= 1 + x + x^2 + x^3 + x^4 + \dots + x^n + o(x^n)$	$= \sum_{k=0}^n x^k + o(x^n)$
$\sqrt{1+x}$	$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \dots + \binom{1/2}{n} x^n + o(x^n)$	$= \sum_{k=0}^n \binom{1/2}{k} x^k + o(x^n)$
$\frac{1}{\sqrt{1+x}}$	$= 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \dots + \binom{-1/2}{n} x^n + o(x^n)$	$= \sum_{k=0}^n \binom{-1/2}{k} x^k + o(x^n)$
$\sqrt[3]{1+x}$	$= 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 + \dots + \binom{1/3}{n} x^n + o(x^n)$	$= \sum_{k=0}^n \binom{1/3}{k} x^k + o(x^n)$
$\frac{1}{\sqrt[3]{1+x}}$	$= 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{7}{81}x^3 + \dots + \binom{-1/3}{n} x^n + o(x^n)$	$= \sum_{k=0}^n \binom{-1/3}{k} x^k + o(x^n)$

Si ricordi che  $\forall \alpha \in \mathbb{R}$  si pone  $\binom{\alpha}{0} = 1$  e  $\binom{\alpha}{n} = \overbrace{\frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}}^{n \text{ fattori}}$  se  $n \geq 1$ .